Research Statement

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My main interests lie in the intersection of topology and quantum computation (QC).

Topologically flavoured mathematics can enter into quantum computation at the hardware level, the software level or the algorithmic level. At the hardware level, many quantum information scientists currently believe that exploiting the 'topological phase' of matter will offer the most promising solution to protecting the delicate quantumness which gives QC its revolutionary power. This approach is usually referred to as "topological QC", and was pioneered by Kitaev, who suggested the use of representations of quantum doubles in QC (quant-ph/9707021), and later gave a categorical description (cond-mat/0506438) in terms of modular tensor categories. Some physical systems are believed to be described by Witten-Chern-Simons theories, and Freedman et al. (quant-ph/0001108) showed that the topological modular functors from these theories are capable of efficiently approximating any QC. At the software level, the essential error-correcting software of a quantum computer can be inspired by topology when the rest of the computer might still be quantum circuit-based. These are known as "topological codes", and the most well-known example is the toric code (quant-ph/0110143), while the most recent example is the Turaev-Viro code (arXiv:1002.2816). At the algorithmic level, we can use a quantum computer to solve topological problems. We call these topological algorithms, for example the algorithm to approximate the Jones polynomial (quant-ph/0511096). Kauffman (arXiv:1001.0354 and personal communications) has recently suggested thinking about quantum algorithms for computing the Khovanov homology. The underlying promise for all these is the ability of QC to efficiently simulate topological quantum field theories argued by Freedman et al. (quant-ph/0001071). These topological algorithms will also help us characterize the complexity of specific topological problems.

My program of research is to try to fully understand the interplay between the three aspects of "topology intersect QC" described above. I think an important piece of the puzzle lies within the (near) equivalence of modular tensor categories, topological quantum field theories and modular functors (book by Bakalov and Kirillov). I would like to reinterpret what all of these mean in the context of QC and ultimately obtain a similar unified picture for "topology intersect QC".

The current project I am working on is joint with Hector Bombin, a postdoc at the Perimeter Institute. We are working on enriching Kitaev's quantum double-based anyon model on a graph by physically introducing a new topological feature, called 'twists' (not the Dehn twists) (arXiv:1004.1838). Given an oriented graph on a (orientable) surface and a finite group, we start by defining various physically meaningful operator algebras acting on a Hilbert space associated to the data (following the approach in arXiv:0712.0190). By studying their representation theory in the presence of twists, we hope to characterize the new underlying algebraic object and thereby understand the topological excitations (or anyon types). After this representation-theoretical analysis, we plan to follow up by providing a more general categorical description of topological twists, extending the theory beyond the limitations of an underlying graph and a finite group.